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Lepton flavor violating Z-boson decays induced by scalar unparticle

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Abstract. We predict the branching ratios of the lepton-flavor violating Z-boson decays $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ in the case that the lepton-flavor violation is carried by scalar unparticle mediation. We observe that their BRs are strongly sensitive to the unparticle scaling dimension, and the branching ratios can reach values of the order of 10^{-8}, for the heavy lepton-flavor case, for small values of the scaling dimension.

Lepton flavor violating (LFV) interactions raised great interest since they are sensitive to physics beyond the standard model (SM) and the related experimental measurements are improved at present. $Z \to l_1 l_2$ decays are among the LFV interactions and theoretical predictions of their branching ratios (BRs) in the framework of the SM are extremely small [1–8]:

$$\mathrm{BR}(Z \to e^{\pm}\mu^{\pm}) \sim \mathrm{BR}(Z \to e^{\pm}\tau^{\pm}) \sim 10^{-54}\,, \quad \mathrm{BR}(Z \to \mu^{\pm}\tau^{\pm}) < 4\times 10^{-60} \tag{1}$$

in the case of non-zero lepton mixing mechanism [9–11]. These results are far from the experimental limits obtained at LEP1 [12]:

$$\begin{aligned} \mathrm{BR}(Z \to e^{\pm}\mu^{\pm}) &< 1.7\times 10^{-6} \quad [13]\,,\\ \mathrm{BR}(Z \to e^{\pm}\tau^{\pm}) &< 9.8\times 10^{-6} \quad [13,14]\,,\\ \mathrm{BR}(Z \to \mu^{\pm}\tau^{\pm}) &< 1.2\times 10^{-5} \quad [13,15]\,, \end{aligned} \tag{2}$$

and from the improved ones at giga-Z [16]:

$$\begin{aligned} \mathrm{BR}(Z \to e^{\pm}\mu^{\pm}) &< 2\times 10^{-9}\,,\\ \mathrm{BR}(Z \to e^{\pm}\tau^{\pm}) &< f\times 6.5\times 10^{-8}\,,\\ \mathrm{BR}(Z \to \mu^{\pm}\tau^{\pm}) &< f\times 2.2\times 10^{-8}\,, \end{aligned} \tag{3}$$

with $f = 0.2$–1.0.[1] On the other hand the giga-Z option of the Tesla project aims to increase the production of Z-bosons at resonance [17]. These numerical values and the forthcoming projects stimulate one to do theoretical work on the LFV Z decays and to enhance their BRs by considering new scenarios beyond the SM. There are various works related to these decays in the literature [1–8, 12–16, 18–25], namely the extension of νSM with one (two) heavy ordinary Dirac (right-handed singlet Majorana) neutrino(s) [7, 8], the Zee model [18], the two Higgs doublet model (2HDM) [19], the 2HDM with extra dimensions [20–22], supersymmetric models [23, 24], and the top-color assisted technicolor model [25].

In the present work, we consider lepton-flavor (LF) violation to be carried by the scalar unparticle (U)-lepton–lepton vertex and unparticles to appear in the internal line, in the loop. The unparticle idea was introduced by Georgi [26, 27] and its effect in the processes, which are induced at least in one loop level, is studied in various works [28–37]. The starting point of this idea is the interaction of the SM and the ultraviolet sector, having a non-trivial infrared fixed point, at the high energy level. The ultraviolet sector comes out as new degrees of freedom, called unparticles, being massless and having non-integral scaling dimension d_u around $\Lambda_U \sim 1$ TeV. The effective lagrangian which drives the interactions of unparticles with the SM fields in the low energy level reads

$$\mathcal{L}_{\mathrm{eff}} \sim \frac{\eta}{\Lambda_U^{d_u+d_{\mathrm{SM}}-n}} O_{\mathrm{SM}} O_U\,, \tag{4}$$

where O_U is the unparticle operator, the parameter η is related to the energy scale of the ultraviolet sector, the low energy one and the matching coefficient [26, 27, 38], and n is the space-time dimension.

At this stage, we choose the appropriate operators in order to drive the LFV decays.[2] The effective interaction

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[1] Notice that these numbers are obtained for the decays $Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1$, where

$$\mathrm{BR}\left(Z \to l_1^{\pm} l_2^{\pm}\right) = \frac{\Gamma\left(Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1\right)}{\Gamma_Z}\,.$$

[2] Notice that the operators with the lowest possible dimension are chosen, since they have the most powerful effect on the low energy effective theory (see for example [39]).

lagrangian responsible for the LFV decays in the low energy effective theory is

$$\mathcal{L}_1 = \frac{1}{\Lambda_U^{du-1}} \left(\lambda^{\rm S}_{ij} \bar{l}_i l_j + \lambda^{\rm P}_{ij} \bar{l}_i {\rm i}\gamma_5 l_j \right) O_U \,, \qquad (5)$$

where l is the lepton field and $\lambda^{\rm S}_{ij}$ ($\lambda^{\rm P}_{ij}$) is the scalar (pseudoscalar) coupling. On the other hand, there is a possibility that a tree level U–Z–Z interaction exists[3] and it has a contribution to the LFV Z decays (see Fig. 1b and c). The corresponding effective Lagrangian reads

$$\mathcal{L}_2 = \frac{\lambda_0}{\Lambda_U^{du}} F_{\mu\nu} F^{\mu\nu} O_U \,, \qquad (6)$$

where $F_{\mu\nu}$ is the field tensor for the Z_μ field and λ_0 is the effective coupling constant.

The one loop level $Z \to l_1 l_2$ decay (see Fig. 1) is carried with the help of the scalar unparticle propagator, which is obtained by using the scale invariance [27, 40]:

$$\int {\rm d}^4x {\rm e}^{{\rm i}px} \langle 0 | T(O_U(x) O_U(0)) 0 \rangle = {\rm i}\frac{A_{d_u}}{2\pi} \int_0^\infty {\rm d}s \frac{s^{d_u-2}}{p^2 - s + {\rm i}\epsilon} = {\rm i}\frac{A_{d_u}}{2\sin(d_u\pi)} (-p^2 - {\rm i}\epsilon)^{d_u-2} \,, \qquad (7)$$

with the factor A_{d_u}

$$A_{d_u} = \frac{16\pi^{5/2}}{(2\pi)^{2d_u}} \frac{\Gamma\left(d_u + \frac{1}{2}\right)}{\Gamma(d_u - 1)\Gamma(2d_u)} \,. \qquad (8)$$

The function $\frac{1}{(-p^2-{\rm i}\epsilon)^{2-d_u}}$ in (7) becomes

$$\frac{1}{(-p^2 - {\rm i}\epsilon)^{2-d_u}} \to \frac{{\rm e}^{-{\rm i}d_u\pi}}{(p^2)^{2-d_u}} \,, \qquad (9)$$

for $p^2 > 0$, and a non-trivial phase appears as a result of the non-integral scaling dimension.

Now, we present the general effective vertex for the interaction of the on-shell Z-boson with a fermionic current:

$$\Gamma_\mu = \gamma_\mu (f_{\rm V} - f_{\rm A}\gamma_5) + \frac{\rm i}{m_W} (f_{\rm M} + f_{\rm E}\gamma_5) \sigma_{\mu\nu} q^\nu \,, \qquad (10)$$

where q is the momentum transfer, $q^2 = (p - p')^2$, $f_{\rm V}$ ($f_{\rm A}$) is the vector (axial-vector) coupling, and $f_{\rm M}$ ($f_{\rm E}$) magnetic (electric) transitions of unlike fermions. Here p ($-p'$) is the four momentum vector of the lepton (anti-lepton). The form factors $f_{\rm V}$, $f_{\rm A}$, $f_{\rm M}$ and $f_{\rm E}$ in (10) are obtained as

$$\begin{aligned} f_{\rm V} &= \int_0^1 {\rm d}x f_{V\,self} + \int_0^1 {\rm d}x \int_0^{1-x} {\rm d}y f_{\rm V\ vert} \,, \\ f_{\rm A} &= \int_0^1 {\rm d}x f_{A\ self} + \int_0^1 {\rm d}x \int_0^{1-x} {\rm d}y f_{\rm A\ vert} \,, \\ f_{\rm M} &= \int_0^1 {\rm d}x \int_0^{1-x} {\rm d}y f_{\rm M\ vert} \,, \\ f_{\rm E} &= \int_0^1 {\rm d}x \int_0^{1-x} {\rm d}y f_{\rm E\ vert} \,. \end{aligned} \qquad (11)$$

Taking into account all the masses of the internal leptons and the external lepton (anti-lepton), the explicit expressions of $f_{\rm V\ self}$, $f_{\rm A\ self}$, $f_{\rm V\ vert}$, $f_{\rm A\ vert}$, $f_{\rm M\ vert}$ and $f_{\rm E\ vert}$ read

$$\begin{aligned} f_{\rm V\ self} = & \frac{c_{\rm self}(1-x)^{1-d_u}}{32 s_{\rm W} c_{\rm W} \pi^2 \left(m^2_{l_2^+} - m^2_{l_1^-}\right)(1-d_u)} \\ & \times \sum_{i=1}^{3} \Big\{ \left(L_{\rm self}^{d_u-1} - L'^{d_u-1}_{\rm self} \right) \\ & \times \Big(m_i \Big((m_{l_1^-} c_2 + m_{l_2^+} c_1) \left(\lambda^{\rm S}_{il_1} + {\rm i}\lambda^{\rm P}_{il_1}\right) \left(\lambda^{\rm P}_{il_2} - {\rm i}\lambda^{\rm S}_{il_2}\right) \\ & - (m_{l_1^-} c_1 + m_{l_2^+} c_2) \left(\lambda^{\rm S}_{il_1} - {\rm i}\lambda^{\rm P}_{il_1}\right) \left(\lambda^{\rm P}_{il_2} + {\rm i}\lambda^{\rm S}_{il_2}\right) \Big) \\ & - m_{l_1^-} m_{l_2^+} (1-x) \Big(c_1 \left({\rm i}\lambda^{\rm P}_{il_1} - \lambda^{\rm S}_{il_1}\right) \left(\lambda^{\rm P}_{il_2} - {\rm i}\lambda^{\rm S}_{il_2}\right) \\ & + c_2 \left({\rm i}\lambda^{\rm P}_{il_1} + \lambda^{\rm S}_{il_1}\right) \left(\lambda^{\rm P}_{il_2} + {\rm i}\lambda^{\rm S}_{il_2}\right) \Big) \Big) \end{aligned}$$

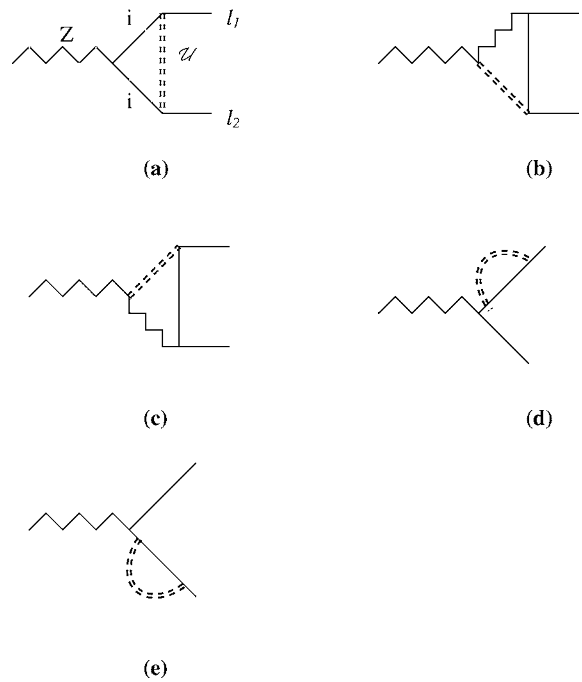

Fig. 1. One loop diagrams contribute to $Z \to l_1^- l_2^+$ decay with scalar unparticle mediator. The *solid line* represents the lepton field: i represents the internal lepton, l_1^- (l_2^+) the outgoing lepton (anti-lepton), the *wavy line* the Z-boson field, the *double dashed line* the unparticle field

[3] The vertex factor is $\frac{4{\rm i}}{\Lambda_U^{du}} \lambda_0 (k_{1\nu} k_{2\mu} - k_1.k_2 g_{\mu\nu})$, where $k_{1(2)}$ is the four momentum of the Z-boson with polarization vector $\epsilon_{1\mu(2\nu)}$.

$$
\begin{aligned}
&+\Big(m_{l_2^+}^2 L'^{d_u-1}_{\text{self}} - m_{l_1^-}^2 L^{d_u-1}_{\text{self}}\Big)(1-x) \\
&\times \Big(c_1 (\mathrm{i}\lambda^{\mathrm{P}}_{il_1} + \lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&+ c_2\,(\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{S}}_{il_2} + \mathrm{i}\lambda^{\mathrm{P}}_{il_2})\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{\text{A self}} = {}& \frac{c_{\text{self}}(1-x)^{1-d_u}}{32 s_{\mathrm{W}} c_{\mathrm{W}} \pi^2 \Big(m_{l_2^+} - m_{l_1^-}\Big)(1-d_u)} \\
&\times \sum_{i=1}^{3} \Big\{ \Big(L^{d_u-1}_{\text{self}} - L'^{d_u-1}_{\text{self}}\Big) \\
&\times \Big(m_i\Big((m_{l_2^+} c_1 - m_{l_1^-} c_2)\,(\lambda^{\mathrm{S}}_{il_1} + \mathrm{i}\lambda^{\mathrm{P}}_{il_1}) \\
&\times (\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) - (m_{l_1^-} c_1 - m_{l_2^+} c_2)\,(\lambda^{\mathrm{S}}_{il_1} - \mathrm{i}\lambda^{\mathrm{P}}_{il_1}) \\
&\times (\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big) - m_{l_1^-} m_{l_2^+} (1-x) \\
&\times \Big(c_1\,(\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\mathrm{i}\lambda^{\mathrm{P}}_{il_2} + \lambda^{\mathrm{S}}_{il_2}) + c_2\,(\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \\
&\times (\lambda^{\mathrm{S}}_{il_2} - \mathrm{i}\lambda^{\mathrm{P}}_{il_2})\Big)\Big) + \Big(m_{l_2^+}^2 L'^{d_u-1}_{\text{self}} - m_{l_1^-}^2 L^{d_u-1}_{\text{self}}\Big) \\
&\times (1-x)\Big(c_1\,(\lambda^{\mathrm{S}}_{il_1} + \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&+ c_2\,(\lambda^{\mathrm{S}}_{il_1} - \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{\text{V vert}} = {}& \frac{-c_{\text{ver}}(1-x-y)^{1-d_u}}{32\pi^2} \sum_{i=1}^{3} \frac{1}{L^{2-d_u}_{\text{vert}}} \Big\{ m_i (1-x-y) \\
&\times \Big((m_{l_2^+} c_1 + m_{l_1^-} c_2)\,(\lambda^{\mathrm{S}}_{il_1} + \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&\times -(m_{l_1^-} c_1 + m_{l_2^+} c_2)\,(\lambda^{\mathrm{S}}_{il_1} - \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big) \\
&- m_i^2 \Big(c_1\,(\lambda^{\mathrm{S}}_{il_1} + \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&+ c_2\,(\mathrm{i}\lambda^{\mathrm{P}}_{il_1} - \lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big) \\
&- \Big(c_1\,(\mathrm{i}\lambda^{\mathrm{P}}_{il_1} - \lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) + c_2\,(\mathrm{i}\lambda^{\mathrm{P}}_{il_1} + \lambda^{\mathrm{S}}_{il_1}) \\
&\times (\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big) \\
&\times \Big(m_Z^2 xy + m_{l_1^-} m_{l_2^+} (1-x-y)^2 - \frac{L_{\text{vert}}}{1-d_u}\Big)\Big\} \\
&- \frac{\lambda_0 m_Z^2}{16\pi^2} \sum_{i=1}^{3} \Big\{ \frac{b_{\text{ver}} y^{1-d_u}}{L^{2-d_u}_{1\ \text{vert}}} \Big\{ (\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&\times (c_1 m_i (1-x+y) + c_2 (m_{l_1^-} x (x+y-1) \\
&+ m_{l_2^+} y (1+x+y))) - (\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&\times (c_2 m_i (1-x+y) + c_1 (m_{l_1^-} x (x+y-1) \\
&+ m_{l_2^+} y (1+x+y)))\Big\} - \frac{b'_{\text{ver}} x^{1-d_u}}{L^{2-d_u}_{2\ \text{vert}}} \Big\{ (\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \\
&\times (c_1 m_i (1+x-y) + c_2 (m_{l_2^+} y (x+y-1) \\
&+ m_{l_1^-} x (1+x+y))) - (\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \\
&\times (c_2 m_i (1+x-y) + c_1 (m_{l_2^+} y (x+y-1) \\
&+ m_{l_1^-} x (1+x+y)))\Big\}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{\text{A vert}} = {}& \frac{-c_{\text{ver}}(1-x-y)^{1-d_u}}{32\pi^2} \sum_{i=1}^{3} \frac{1}{L^{2-d_u}_{\text{vert}}} \Big\{ m_i (1-x-y) \\
&\times \Big((m_{l_2^+} c_1 - m_{l_1^-} c_2)\,(\lambda^{\mathrm{S}}_{il_1} + \mathrm{i}\lambda^{\mathrm{P}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&- (m_{l_2^+} c_2 - m_{l_1^-} c_1)\,(\mathrm{i}\lambda^{\mathrm{P}}_{il_1} - \lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big) \\
&+ m_i^2 \Big(c_1\,(\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{S}}_{il_2} - \mathrm{i}\lambda^{\mathrm{P}}_{il_2}) \\
&+ c_2\,(\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{S}}_{il_2} + \mathrm{i}\lambda^{\mathrm{P}}_{il_2})\Big) \\
&+ \Big(c_1\,(\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{S}}_{il_2} + \mathrm{i}\lambda^{\mathrm{P}}_{il_2}) \\
&+ c_2\,(\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{S}}_{il_2} - \mathrm{i}\lambda^{\mathrm{P}}_{il_2})\Big) \\
&\times \Big(m_Z^2 xy - m_{l_1^-} m_{l_2^+} (1-x-y)^2 - \frac{L_{\text{vert}}}{1-d_u}\Big)\Big\} \\
&+ \frac{\lambda_0 m_Z^2}{16\pi^2} \sum_{i=1}^{3} \Big\{ \frac{b_{\text{ver}} y^{1-d_u}}{L^{2-d_u}_{1\ \text{vert}}} \Big\{ (\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&\times \Big(c_1 m_i (x-y-1) + c_2 (m_{l_1^-} x (1-x-y) \\
&+ m_{l_2^+} y (1+x+y))\Big) + (\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) \\
&\times \Big(c_2 m_i (x-y-1) + c_1 (m_{l_1^-} x (1-x-y) \\
&+ m_{l_2^+} y (1+x+y))\Big)\Big\} \\
&+ \frac{b'_{\text{ver}} x^{1-d_u}}{L^{2-d_u}_{2\ \text{vert}}} \Big\{ (\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \Big(c_1 m_i (1+x-y) \\
&+ c_2 (m_{l_2^+} y (x+y-1) - m_{l_1^-} x (1+x+y))\Big) \\
&+ (\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \Big(c_2 m_i (1+x-y) + c_1 \\
&\times (m_{l_2^+} y (x+y-1) \\
&- m_{l_1^-} y (1+x+y))\Big)\Big\}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{\text{M vert}} = {}& \frac{-\mathrm{i}(1-x-y)^{1-d_u}}{32\pi^2} \sum_{i=1}^{3} \frac{c_{\text{ver}} m_Z c_{\mathrm{W}}}{L^{2-d_u}_{\text{vert}}} \Big\{ m_i \Big((x+y) \\
&\times (\lambda^{\mathrm{S}}_{il_1} \lambda^{\mathrm{S}}_{il_2} - \lambda^{\mathrm{P}}_{il_1} \lambda^{\mathrm{P}}_{il_2})\,(c_1 + c_2) - \mathrm{i}(x-y) \\
&\times (\lambda^{\mathrm{S}}_{il_1} \lambda^{\mathrm{P}}_{il_2} + \lambda^{\mathrm{P}}_{il_1} \lambda^{\mathrm{S}}_{il_2})\,(c_2 - c_1)\Big) \\
&+ (1-x-y)(m_{l_1^-} x + m_{l_2^+} y) \Big(c_1\,(\lambda^{\mathrm{P}}_{il_1} + \mathrm{i}\lambda^{\mathrm{S}}_{il_1}) \\
&\times (\lambda^{\mathrm{P}}_{il_2} - \mathrm{i}\lambda^{\mathrm{S}}_{il_2}) + c_2\,(\lambda^{\mathrm{P}}_{il_1} - \mathrm{i}\lambda^{\mathrm{S}}_{il_1})\,(\lambda^{\mathrm{P}}_{il_2} + \mathrm{i}\lambda^{\mathrm{S}}_{il_2})\Big)\Big\} \\
&- \frac{\mathrm{i}\lambda_0}{16\pi^2} \sum_{i=1}^{3} \Big\{ \frac{b_{\text{ver}} m_Z c_{\mathrm{W}} y^{1-d_u}}{L^{2-d_u}_{1\ \text{vert}}} \Big(\Big(c_1\,(\lambda^{\mathrm{S}}_{il_2} + \mathrm{i}\lambda^{\mathrm{P}}_{il_2}) \\
&+ c_2 (\lambda^{\mathrm{S}}_{il_2} - \mathrm{i}\lambda^{\mathrm{P}}_{il_2})\Big)\Big(2 m_Z^2 xy + (1-x-y) \\
&\times \Big(m_{l_1^-}^2 x + m_{l_2^+}^2 y - m_{l_1^-} m_{l_2^+} (x+y)\Big) - 2\frac{L_{1\ \text{vert}}}{1-d_u}\Big) \\
&- \Big(c_1\,(\lambda^{\mathrm{S}}_{il_2} - \mathrm{i}\lambda^{\mathrm{P}}_{il_2}) + c_2\,(\lambda^{\mathrm{S}}_{il_2} + i\lambda^{\mathrm{P}}_{il_2})\Big)(1-x-y) \\
&\times m_i (m_{l_1^-} - m_{l_2^+})\Big) + \frac{b'_{\text{ver}} m_Z c_{\mathrm{W}} x^{1-d_u}}{L^{2-d_u}_{2\ \text{vert}}}
\end{aligned}
$$

$$
\begin{aligned}
&\times\Big(\Big(c_1\left(\lambda^{\rm S}_{il_1}-{\rm i}\lambda^{\rm P}_{il_1}\right)+c_2\left(\lambda^{\rm S}_{il_1}+{\rm i}\lambda^{\rm P}_{il_1}\right)\Big)\\
&\times\Big(2m_Z^2xy+(1-x-y)\Big(m^2_{l_1^-}x+m^2_{l_2^+}y-m_{l_1^-}m_{l_2^+}\\
&\times(x+y)\Big)-2\frac{L_{2\,\rm vert}}{1-d_u}\Big)\\
&+\Big(c_1\left(\lambda^{\rm S}_{il_1}+{\rm i}\lambda^{\rm P}_{il_1}\right)+c_2\left(\lambda^{\rm S}_{il_1}-{\rm i}\lambda^{\rm P}_{il_1}\right)\Big)\\
&\times(1-x-y)m_i(m_{l_1^-}-m_{l_2^+})\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{\rm E\ vert}=&\frac{-{\rm i}(1-x-y)^{1-d_u}}{32\pi^2}\sum_{i=1}^{3}\frac{c_{\rm ver}m_Z c_{\rm W}}{L_{\rm vert}^{2-d_u}}\Big\{m_i\\
&\times\Big({\rm i}(x+y)\left(\lambda^{\rm S}_{il_1}\lambda^{\rm P}_{il_2}+\lambda^{\rm P}_{il_1}\lambda^{\rm S}_{il_2}\right)(c_1+c_2)\\
&+(x-y)\left(\lambda^{\rm P}_{il_1}\lambda^{\rm P}_{il_2}-\lambda^{\rm S}_{il_1}\lambda^{\rm S}_{il_2}\right)(c_2-c_1)\Big)\\
&+(1-x-y)(m_{l_1^-}x-m_{l_2^+}y)\Big(c_1\left(\lambda^{\rm P}_{il_1}+{\rm i}\lambda^{\rm S}_{il_1}\right)\\
&\times(\lambda^{\rm P}_{il_2}-{\rm i}\lambda^{\rm S}_{il_2})-c_2\left(\lambda^{\rm P}_{il_1}-{\rm i}\lambda^{\rm S}_{il_1}\right)\left(\lambda^{\rm P}_{il_2}+{\rm i}\lambda^{\rm S}_{il_2}\right)\Big)\Big\}\\
&-\frac{{\rm i}\lambda_0}{16\pi^2}\sum_{i=1}^{3}\Big\{\frac{b_{\rm ver}m_Z c_{\rm W}y^{1-d_u}}{L_{1\,\rm vert}^{2-d_u}}\Big(\Big(c_1\left(\lambda^{\rm S}_{il_2}+{\rm i}\lambda^{\rm P}_{il_2}\right)\\
&-c_2(\lambda^{\rm S}_{il_2}-{\rm i}\lambda^{\rm P}_{il_2})\Big)\Big(2m_Z^2xy+(1-x-y)\\
&\times\Big(m^2_{l_1^-}x+m^2_{l_2^+}y+m_{l_1^-}m_{l_2^+}(x+y)\Big)-2\frac{L_{1\,\rm vert}}{1-d_u}\Big)\\
&+\Big(c_1(\lambda^{\rm S}_{il_2}-{\rm i}\lambda^{\rm P}_{il_2})-c_2(\lambda^{\rm S}_{il_2}+{\rm i}\lambda^{\rm P}_{il_2})\Big)\\
&\times(1-x-y)m_i(m_{l_1^-}+m_{l_2^+})\Big)\\
&-\frac{b'_{\rm ver}m_Z c_{\rm W}x^{1-d_u}}{L_{1\,\rm vert}^{2-d_u}}\Big(\Big(c_1(\lambda^{\rm S}_{il_1}-{\rm i}\lambda^{\rm P}_{il_1})\\
&-c_2(\lambda^{\rm S}_{il_1}+{\rm i}\lambda^{\rm P}_{il_1})\Big)\Big(2m_Z^2xy+(1-x-y)\\
&\Big(m_{l_1^-}x+m^2_{l_2^+}y+m_{l_1^-}m_{l_2^+}(x+y)\Big)-2\frac{L_{2\,\rm vert}}{1-d_u}\Big)\\
&+\Big(c_1(\lambda^{\rm S}_{il_1}+{\rm i}\lambda^{\rm P}_{il_1})-c_2\left(\lambda^{\rm S}_{il_1}-{\rm i}\lambda^{\rm P}_{il_1}\right)\Big)(1-x-y)\\
&\times m_i\left(m_{l_1^-}+m_{l_2^+}\right)\Big)\Big\}, \qquad (12)
\end{aligned}
$$

with

$$
\begin{aligned}
L_{\rm self}&=x\left(m^2_{l_1^-}(1-x)-m_i^2\right),\\
L'_{\rm self}&=x\left(m^2_{l_2^+}(1-x)-m_i^2\right),\\
L_{\rm vert}&=\left(m^2_{l_1^-}x+m^2_{l_2^+}y\right)(1-x-y)-m_i^2(x+y)+m_Z^2xy,\\
L_{1\,\rm vert}&=\left(m^2_{l_1^-}x+m^2_{l_2^+}y-m_i^2\right)(1-x-y)+m_Z^2x(y-1),\\
L_{2\,\rm vert}&=\left(m^2_{l_1^-}x+m^2_{l_2^+}y-m_i^2\right)(1-x-y)+m_Z^2y(x-1),
\end{aligned}
\qquad (13)
$$

and

$$
\begin{aligned}
c_{\rm self}&=-\frac{eA_{d_u}}{2\sin(d_u\pi)\Lambda_u^{2(d_u-1)}},\\
c_{\rm ver}&=-\frac{eA_{d_u}}{2s_{\rm W}c_{\rm W}\sin(d_u\pi)\Lambda_u^{2(d_u-1)}},\\
b_{\rm ver}&=-\frac{eA_{d_u}}{2s_{\rm W}c_{\rm W}\sin(d_u\pi)\Lambda_u^{2d_u-1}},\\
b'_{\rm ver}&=-b_{\rm ver}\,.
\end{aligned}
\qquad (14)
$$

In (12), the flavor changing scalar and pseudoscalar couplings $\lambda^{\rm S,P}_{il_{1(2)}}$ represent the effective interaction between the internal lepton i ($i=e,\mu,\tau$) and the outgoing l_1^- (l_2^+) lepton (anti-lepton). Finally, BR for $Z\to l_1^-l_2^+$ can be obtained by using the form factors $f_{\rm V}$, $f_{\rm A}$, $f_{\rm M}$ and $f_{\rm E}$:

$$
{\rm BR}(Z\to l_1^-l_2^+)=\frac{1}{48\pi}\frac{m_Z}{\Gamma_Z}\Big\{|f_{\rm V}|^2+|f_{\rm A}|^2+\frac{1}{2\cos^2\theta_{\rm W}}(|f_{\rm M}|^2+|f_{\rm E}|^2)\Big\}, \qquad (15)
$$

where Γ_Z is the total decay width of the Z-boson. Note that, in general, the production of sum of charged states is considered with the corresponding BR

$$
{\rm BR}(Z\to l_1^\pm l_2^\pm)=\frac{\Gamma(Z\to(\bar l_1 l_2+\bar l_2 l_1))}{\Gamma_Z}, \qquad (16)
$$

and in our numerical analysis we use this branching ratio.

1 Discussion

In this section, we estimate the BRs of LFV Z-boson decays by considering flavor violation to be carried by scalar unparticle mediation. These decays exist at least in one loop level and, in the present case, we assume that the possible sources of LF violation are the U–lepton–lepton couplings in the framework of the effective theory. On the other hand, we take the U–Z–Z coupling to be non-zero and we study the sensitivity of the BRs to this coupling. The couplings considered and the scaling dimension of the unparticle(s) are free parameters, and they should be restricted by respecting the current experimental measurements and some theoretical considerations. For the scaling dimension d_u we choose the range[4] $1<d_u<2$. For the LF violating couplings we consider the following restrictions.

– The (off-) diagonal couplings are flavor (blind and universal) aware and $\lambda_{\tau\tau}>\lambda_{\mu\mu}>\lambda_{ee}$ ($\lambda_{ij}, i\neq j$). We take the greatest numerical value of the diagonal coupling to be of the order of 1 and the off-diagonal one as $\lambda_{ij}=\kappa\lambda_{ee}$ with $\kappa<1$. In our numerical calculations, we choose $\kappa=0.5$.

[4] Here, $d_u>1$ is due to the non-integrable singularities in the decay rate [27] and $d_u<2$ is due to the convergence of the integrals [30].

Parameter	Value
m_e	0.0005 GeV
m_μ	0.106 GeV
m_τ	1.780 GeV
Γ_Z^{Tot}	2.49 GeV
s_W^2	0.23

Table 1. The values of the input parameters used in the numerical calculations

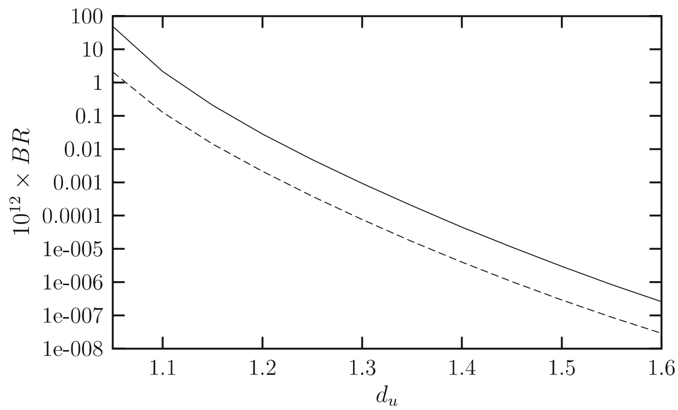

Fig. 2. The scale parameter d_u dependence of $\mathrm{BR}(Z \to \mu^\pm e^\pm)$ for $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the *solid (dashed) line* represents the BR for the total contribution and $\lambda_0 = 0.1$ (the contribution due to the U–Z–Z vertex and $\lambda_0 = 1$)

– As a second possibility, we consider the (off-) diagonal couplings to be flavor blind and universal and of the order of 1. Similar to the previous case, we take the off-diagonal ones as $\lambda_{ij} = \kappa \lambda_{ii}$ with $\kappa = 0.5$.

Furthermore, we choose the coupling λ_0 for the tree level U–Z–Z interaction (see (6)) in the range 0.1–1.0 and we take the energy scale to be of the order of TeV. Notice that throughout our calculations we use the input values given in Table 1.

In Fig. 2, we present the $\mathrm{BR}(Z \to \mu^\pm e^\pm)$, with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the U–Z–Z vertex and $\lambda_0 = 1$). The BR is strongly sensitive to the scale d_u and reaches numerical values 10^{-10}, for $d_u < 1.1$. The contribution of the U–Z–Z vertex is almost two orders smaller than the total one, even for $\lambda_0 = 1$. With increasing values of the scaling dimension d_u, the BR sharply decreases and becomes negligible. Figure 3 represents $\mathrm{BR}(Z \to \mu^\pm e^\pm)$, with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed and small dashed) line represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10$ TeV (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0$ TeV with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10$ TeV). In the case that the

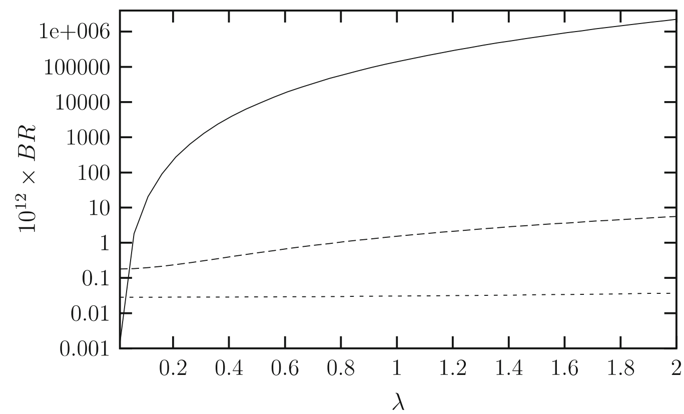

Fig. 3. $\mathrm{BR}(Z \to \mu^\pm e^\pm)$ with respect to the couplings λ, for $d_u = 1.2$. Here the *solid (dashed, small dashed) line* represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10$ TeV (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0$ TeV, with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10$ TeV)

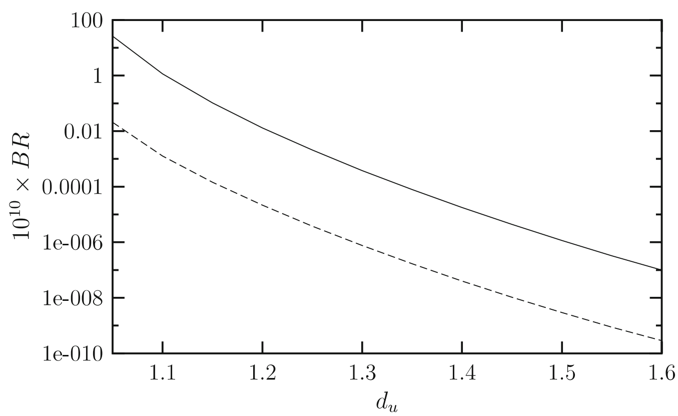

Fig. 4. The scale parameter d_u dependence of $\mathrm{BR}(Z \to \tau^\pm \mu^\pm)$ for $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the *solid (dashed) line* represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the U–Z–Z vertex and $\lambda_0 = 1$)

diagonal (off-diagonal) couplings are flavor blind, the BR may reach values of the order of 10^{-7} for $\lambda = 1$. This may ensure valuable information about unparticle physics and the LFV couplings with more accurate measurements of the decays under consideration. Furthermore, this figure shows that the BR is not sensitive to the coupling λ_0, and it is enhanced almost one order in the range $0.1 \leq \lambda_0 \leq 1.0$, for the energy $\Lambda_u = 1.0$ TeV.

Figure 4 shows $\mathrm{BR}(Z \to \tau^\pm \mu^\pm)$[5] with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for the total contribution and $\lambda_0 = 0.1$ (the contribution due to the U–Z–Z vertex and $\lambda_0 = 1$). The BR is enhanced

[5] For $\mathrm{BR}(Z \to \tau^\pm e^\pm)$ we get almost the same results and we do not present the corresponding figures.

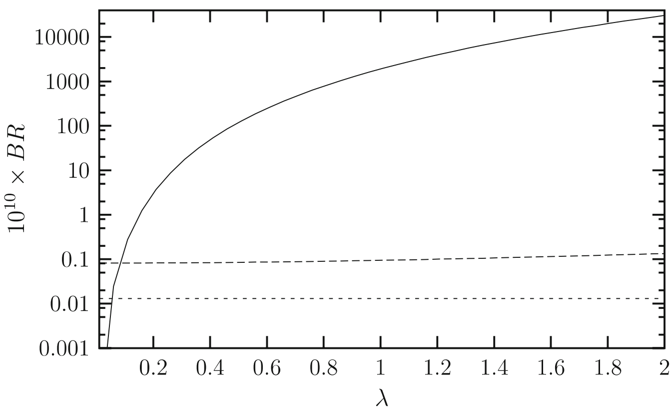

Fig. 5. BR($Z \to \tau^\pm \mu^\pm$) with respect to the couplings λ, for $d_u = 1.2$. Here the *solid (dashed, small dashed) line* represents the BR with respect to λ for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10$ TeV (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0$ TeV, with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10$ TeV)

up to values of the order of 10^{-8}, for $d_u < 1.1$ and increasing values of the scaling dimension d_u result in a considerable suppression in the BR. The contribution due to the U–Z–Z vertex is more than two orders smaller than the total one for $\lambda_0 = 1$, and it shows that the effect of the U–Z–Z vertex becomes weaker for heavy flavor outputs. In Fig. 5, we present BR($Z \to \tau^\pm \mu^\pm$) with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed and small dashed) line represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10$ TeV (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0$ TeV with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$ and $\Lambda_u = 10$ TeV). For the flavor blind diagonal (off-diagonal) couplings, the BR may reach values of the order of 10^{-6} for $\lambda = 1$. On the other hand the BR is not sensitive to the coupling λ_0 and, for the energy $\Lambda_u = 1.0$ TeV, its numerical value is almost one order greater than the one for $\Lambda_u = 10$ TeV.

As a summary, the LFV Z-boson decays are strongly sensitive to the unparticle scaling dimension d_u and, for small values, $d_u < 1.1$, there is a considerable enhancement in the BR. In the case that the diagonal (off-diagonal) couplings are flavor blind and of the order of 1, the BR may reach values of the order of 10^{-6} (10^{-7}) for the decay $Z \to \tau^\pm l^\pm$, $l = \mu$ or e ($Z \to \mu^\pm e^\pm$). With the forthcoming more accurate measurements of the decays under consideration it will be possible to test possible signals coming from the new physics which drives the flavor violation, which here is unparticle physics.

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